SPINNING MAGNETIC FIELDS

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BRIEF SUMMARY

A possible electrical charge model based on the spinning time invariant point magnetic dipole within the framework of classical physics is outlined, as suggested by the admissible circular trajectory of a test charge around the magnetic dipole in its equatorial plane. The model depends on the moving force line hypothesis which has been claimed to have been disproved. The controversy about that issue at the turn of this century is reanalyzed. The mathematical model of the generalized homopolar generator is presented, which is fully in agreement with all experimental facts concerning the homopolar generator, except for a single result by Pegram which appears to be an error and which violates the Faraday law. It is shown that the moving force line hypothesis was apparently not disproved by Kennard as claimed. Thus the intriguing model of electrical charge is possible. Further experimental research with all kinds of spinning magnets appears to be justified and desirable in view of the potentially high significance of the research line initiated by Jehle with the quantum mechanical model of the charge in terms of spinning flux loop forms.

INTRODUCTION

The movement of the lines of force of a magnetic field whose source is rotating represents a very old puzzle dating back to Faraday's discovery of the homopolar generator or the Faraday disk, as it is also referred to in the literature.

The fact is incontestable that the lines of force of a magnetic field do move with the movement of the source of that magnetic field, as long as $\partial\mathbf{B}/\partial t$ is not equal to zero in the coordinate system in which the movement of the source is observed and in which the field $\mathbf{B}$ is defined. This fact can be easily proved experimentally.

However, if there is the axial symmetry of a steady magnetic field and its source, and if that source is rotated around that axis of symmetry with the constant angular velocity, it may appear as though the lines of force do not rotate with the rotation of the source, and that such a rotation is not detectable electromagnetically since $\partial\mathbf{B}/\partial t\equiv\mathbf{0}$ in such a case. This problem arose directly in connection with the homopolar generator. The numerous attempts to resolve that problem led to a controversy, which is described in several papers,[1-5] at the beginning of this century. The additional references to the very voluminous and often very contradictory literature on the homopolar generator can be found in those papers.

A quantum-mechanical electrical charge model based on the spinning flux loop forms was recently proposed and extensively analyzed by Jehle.[6,7] Independently, the present author proposed an electrical charge model based on the spinning point magnetic dipole strictly within the framework of classical physics. It is in connection with that model that the old puzzle of the moving or the nonmoving force line hypothesis surfaced once again.

The objectives of this paper are to indicate the possibility of a new model of the electrical charge and to present a new mathematical model of the so-called generalized homopolar generator in order to discuss the old but apparently still unresolved problem of the moving force line hypothesis or theory.

POINT MAGNETIC DIPOLE

Consider a point magnetic dipole located at the coordinate origin, whose magnetic induction density field is given by

$$\mathbf{B}=\frac{\mu_0}{4\pi}\left(-\frac{\mathbf{m}}{r^3}+\frac{3(\mathbf{m}\cdot\mathbf{r})\,\mathbf{r}}{r^5}\right), \tag{1}$$

where the notation is conventional. The rationalized MKS system of units is utilized. The magnetic dipole moment m is assumed to be constant and directed along the *z* axis.

Let a point charge q of mass m_0 move in the field with velocity v, whose magnitude is assumed to be much smaller than c so that the variation of mass with velocity can be neglected.

The equation of motion is

$$m_0\mathbf{a}=q\mathbf{v}\times\mathbf{B}, \tag{2}$$

where B is given by Eq. (1).

This equation of motion permits the circular trajectory to be a possible solution if for a constant w the velocity vector v is in the equatorial plane,

$$\mathbf{v}=\omega r\,\mathbf{a}_\phi, \tag{3}$$

so that

$$m_0v^2/r=m_0\omega^2r=q\mu_0\omega m/4\pi r^2. \tag{4}$$

The possible circular trajectory of a point electrical charge around a magnetic dipole as specified above is apparently observed as ring currents associated with the geomagnetic storms.

The right-hand side of Eq. (4) can be written

$$q\mu_0\omega m/4\pi r^2=q\mu_0\epsilon_0\omega m/4\pi\epsilon_0 r^2=qQ/4\pi\epsilon_0 r^2, \tag{5}$$

with

$$Q=\mu_0\epsilon_0\omega m=\omega m/c^2. \tag{6}$$

It is clear from the above that the observer can interpret Eq. (5) as though the point charge q revolves around another point charge - Q in the equatorial plane, i. e., under the conditions specified the point magnetic dipole appears as a point charge and is absolutely indistinguishable from a point charge.

This simple but admissible solution of the rather very complex dynamic problem raises the question: What happens if it is assumed that the magnetic dipole itself rotates with the same constant angular velocity ω ?

First of all, the argument that the rotation of a point magnetic dipole, i.e., a point structure, is physically meaningless must be immediately dismissed. The point magnetic dipole is a mathematical category which is used to describe and/or approximate the observed phenomena in nature. It stands for some real objects or particles possessing the magnetic property. It can be rotated by definition.

The answer to the above question appears to be a puzzle. Namely, the magnetic field given by the expression (1) represents, in view of the assumed axial symmetry, a continuum of lines which are not discernible mathematically in any way whatsoever when the transformation is made from one system to the other having the common coordinate origin and the common z axis and rotating with respect to each other around the axis of symmetry, i. e., the z axis.

Nevertheless, it appears that there are two logically possible answers to the above question.

(i) There is no change of the circular trajectory of the test point electrical charge due to the rotation of the magnetic dipole, i. e., the rotation of the magnetic dipole around its axis of symmetry is of no consequence whatsoever and is not detectable by any electromagnetic means. In other words, the magnetic field is not carried by the rotating magnetic dipole. This is the non-moving force line hypothesis or theory, or the N hypothesis for short. It consistently implies that the magnetic field of that axially symmetrical magnetic dipole is an abstraction rather than a physical reality.

(ii) The magnetic dipole carries its own magnetic field with the rotation and, as a consequence, the rotating test charge no longer is moving with respect to the magnetic field, since the magnetic dipole and the test charge rotate together by assumption. In such a case the centrifugal acceleration on the rotating test charge is observed only by a stationary observer and must be balanced by some other force. This second alternative is referred to as the moving force line hypothesis or theory, or the M hypothesis for short.

Purely logically, it may be argued that there is an infinite number of possibilities between those two limiting alternatives. The problem itself is sufficiently puzzling with only two hypotheses, so only those two hypotheses will be analyzed.

POSSIBLE ELECTRICAL CHARGE MODEL

The M hypothesis admits the following electrical charge model. If the magnetic dipole spins with the constant angular velocity $\omega=\omega a_z$, then the stationary test charge in the stationary coordinate system S experiences the electric field by assumption

$$\mathbf{E}=-(\boldsymbol{\omega}\times\mathbf{r})\times\mathbf{B}, \qquad (7)$$

with the minus sign as appropriate and **B** given by Eq. (1), since **B** due to the assumed absence of the ϕ dependence remains the same in all systems having the common z axis and the common coordinate origin.

Introducing Eq. (1) into Eq. (7), the electric field **E** becomes

$$\mathbf{E} = + (\mu_0 \omega m / 4\pi r^2)[(\sin^2\theta)\, \mathbf{a}_r - (2 \sin\theta \cos\theta)\, \mathbf{a}_\theta] , \tag{8}$$

where the notation is conventional for the spherical coordinate system.

It is easy to show that

$$\mathbf{E} = - \nabla U , \tag{9}$$

with

$$U = \mu_0 \omega m \sin^2\theta / 4\pi r . \tag{10}$$

It is clear from the above expressions that the electric field observed by the stationary observer subject to the M hypothesis is strictly conservative by definition, although it is dynamic and magnetic according to its origin, It is not the Coulomb-type field, except in the equatorial plane. Another puzzling or interesting feature of the electric field given by the expression (8) is that its divergence is *not* equal to zero anywhere in the space, which Sommerfeld refers to as "an interesting mathematical difficulty" (cf. Ref. 8, p. 363). In the equatorial plane the electric field may be considered as due to the electrical charge Q given by Eq. (6), with the positive sign. Thus, subject to the validity of the M hypothesis, the spinning point magnetic dipole appears as a charge + Q as far as a stationary test charge in the equatorial plane is concerned. This is in full symmetry with the fact that for a test charge circling around a stationary point magnetic dipole in the equatorial plane the magnetic dipole appears as a charge — Q. Actually, the observer attached to that revolving test charge must observe that the magnetic dipole is circling around the test charge, and simultaneously the magnetic dipole is spinning with the synchronous angular velocity in the negative direction, hence there is the change of sign. There is a perfect symmetry between those two observations if the validity of the M hypothesis is assumed, and this is a strong argument in favor of the M hypothesis. The change of sign is the familiar feature from the homopolar generator.

It should be noted that the force exerted upon the circling point electrical charge by the magnetic dipole is presumably well understood, i. e., the Lorentz force given by Eq, (2). However, under the conditions specified, there is no force whatsoever upon the magnetic dipole due to the circulating charge, and Newton's third law of equality of action and reaction appears to be violated unless the spinning magnetic dipole spinning in the negative mathematical direction relative to the electrical charge is equivalent to the electrical charge. If and only if the spinning magnetic dipole is equivalent to the electrical charge, Newton's third law is not violated in this case. This argument appears to strongly favor the M hypothesis. If, on the other hand, the spinning magnetic dipole is not equivalent to the electrical charge, then this case of the interaction between the magnetic dipole and the electrical charge represents not only the breakdown of Newton's third law, but also the breakdown of the classical electromagnetic field theory.

In order to define the equivalent charge associated with the electric field [Eq. (8)], the calculation of the flux of that electric field over any concentric sphere having its center at the coordinate origin may be used, which yields

$$Q_{eq} = 2\omega m / 3c^2 = \tfrac{2}{3} Q \ . \tag{11}$$

If m is assumed to be the Bohr magneton 9.27×10^{-24} Am^2, and if Q_{eq} is equated to the quantum (electron) charge of 1.6×10^{-19} C, then Eq. (11) yields for the magnitude of the angular velocity

$$\omega = 2.35 \times 10^{21} \ \mathrm{rad/s} \ . \tag{12}$$

This is the possible charge model which exhibits the 6 dependence (axial dependence). For a macroscopic charge distribution involving a very large number of such model charges with different random orientations of axes, the averaging over-all possible orientations may lead to the Coulomb field. The axial dependence may offer the possibility to account for the very intriguing "holes" within the framework of classical physics.

It should be mentioned that Jehle[6,7] defines his quantum-mechanical charge model by averaging over all possible orientations of the axis of the model itself. Jehle's flux loop form corresponds fully to the point magnetic dipole. The averaging used by Jehle is inadmissible within classical physics by definition.

The angular velocity [Eq. (12)] yields for the maximum circular velocity of the electron model a value which is well below c, if the classical electron radius of 2.817×10^{-15} m is used.

There is a puzzling feature of this possible electrical charge model. Namely, the electric field [Eq. (8)] is magnetic according to its origin. As such, it cannot presumably be screened by a non-ferromagnetic metallic conductor. But a true electrostatic field can be screened by metallic conductors, which is the proof of the Coulomb law. A large number of the above-described charge models with random axial orientations unquestionably yields the Coulomb-type field which must be screenable, but it remains obscure how and why. Nevertheless, Jehle's model, as well as the possible charge model outlined above, represent intriguing possibilities which should be explored in spite of some difficulties. This conclusion means that it is worthwhile to reex-amine the problem of the M or the N hypothesis. Namely, Kennard[3,4] claimed that the M hypothesis was experimentally disproved, while Barnett[1,2] maintained that the M hypothesis was not at all disproved. The controversy died in view of the preoccupation of the physicists of that era with Bohr's revolutionary ideas, and Ken-nard's claim was accepted because the physicists were tired of that problem. It will be shown in this paper that all experimental results are fully consistent with either the M or the N hypothesis.

HOMOPOLAR GENERATOR

The homopolar generator in its classical form as discovered and described by Faraday consists of an axially symmetrical permanent magnet, which is rotated around its axis of symmetry, while a stationary metallic wire is connected to two brushes sliding against two different points of the rotating magnet (see Fig. 1). The equatorial point and the end point of the magnet are the typical points for the brushes to be placed in order to obtain the largest possible electrical current, which is observed in the described circuit. If the magnet is stopped while the wire is rotated around the same axis with the same constant angular velocity, but in the opposite direction, the same steady electrical current is observed flowing through that circuit. It is assumed that the permanent magnet is made of an electrically conducting material, thus forming the part of the closed electrical circuit. If a steady electromagnet is used instead of the permanent magnet, then an appropriate coaxial conductor, insulated from the windings of that electromagnet, must cover the electromagnet and rotate with it.

The question now arises: What is the explanation of the phenomenon and which part of the circuit is the seat of the induced electromotive force (EMF), the rotating magnet or the stationary wire?

In order to answer that question, the moving and the nonmoving force line theories or hypotheses have been put forward. The M hypothesis assumes that the rotating magnet carries its own magnetic induction density field, which rotates with the magnet together. The M theory leads obviously to the conclusion that the stationary wire is the seat of the induced EMF and not the rotating magnet. Of course, if the magnet is stationary while the wire rotates the problem disappears, since in that case the wire is obviously the seat of the induced EMF. The second hypothesis, i. e., the N hypothesis, assumes that the rotating magnet does not carry its own magnetic field during the rotation because of the axial symmetry. The N hypothesis leads to the conclusion that it is the rotating magnet which is the seat of the induced EMF, i. e., the rotating magnet cuts the lines of its own but standing magnetic field.

The above question is somewhat oversimplified, since the classical form of the homopolar generator is also oversimplified. Namely, in the case of the steady electromagnet instead of the permanent magnet, it has been already mentioned that a separate coaxial conductor must be present, which must be electrically insulated from the windings of that electromagnet. The coaxial conductor may be a cylinder, a disk, a frustum (cone), or any other coaxial form. The disk with the two brushes attached to two points with different radii is a configuration which is referred to as the Faraday disk, and is quite simple and straightforward. The disk can be easily arranged to rotate independently from the magnet around the same axis of symmetry, but with a different constant angular velocity. As a matter of fact, from the electrical point of view the disk is an axially symmetrical structure which is fully defined by the impedance $Z_1 = Z_1(s)$ offered by the disk at the points of the two brushes, denoted as brushes A and B. The variable s is the familiar complex frequency of the Laplace transform.

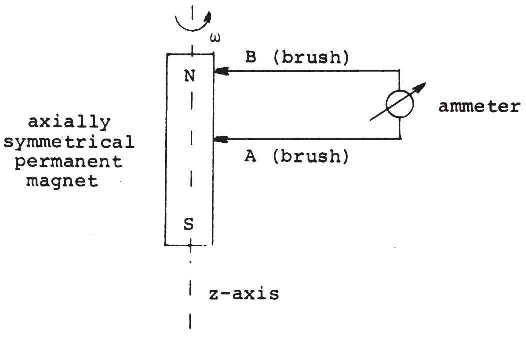

FIG. **1.** Classical homopolar generator.

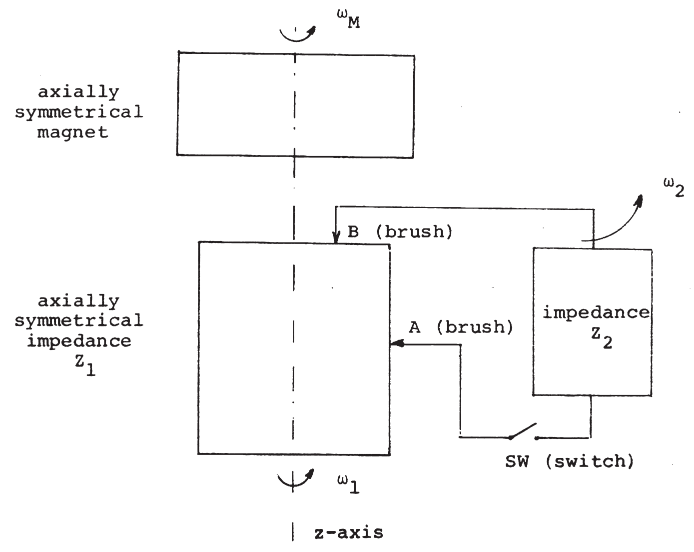

FIG. **2**. Generalized homopolar generator. Note, the impedances Z_1 and Z_2 may be located inside the magnet.

It should be mentioned that in the experimental measurements by Barnett,[1,2] and Kennard,[3,4] coaxial ca-pacitive structures within the almost uniform coaxial magnetic field were used as parts of what was essentially a homopolar generator. Kennard's approach, reported in Ref. 4, with a rotating coaxial capacitor and a stationary electrometer, i. e., stationary capacitor, leads in particular to the concept of the generalized homopolar generator which is defined here as a natural outgrowth of the experimental setups used by Barnett[1] and Kennard.[4]

The impedance Z_1 is almost purely real (resistive) in the case of a disk. Experiments by Kennard[4] and other physicists (viz., Ref. 2, p. 325) unmistakably show that a coaxial capacitor rotating in a magnetic field whose direction coincides with the axis of rotation is a seat of an induced EMF, since such a rotating capacitor can charge a stationary capacitor, i.e., an electrometer. This is a very important experimental result since for the general case of the homopolar generator the disk can be substituted by a coaxial capacitor of the appropriate length, in which case $\boldsymbol{Z}_1$ is approximately equal to $\mathbf{l}/s\boldsymbol{C}_1$, where C_1 is the capacitance of that coaxial capacitor, hence neglecting the small resistances and inductances of the leads and the brushes as well as the large but inevitable leakage resistances of the insulators carrying the armatures of that coaxial capacitor.

Thus, the generalized homopolar generator which is very suitable for the analysis is formed as follows and is sketched in Fig. 2. Let the axis of symmetry of the steady magnet be the z axis. The steady magnet rotates around the z axis with the constant angular velocity $\omega_M \mathrm{a}_z$. Let the axially symmetrical structure characterized electrically by and called the impedance Z_1 rotate around the z axis with the constant angular velocity $\boldsymbol{\omega}_{\mathrm{M}}= \boldsymbol{\omega}_1\mathbf{a}_z$. The impedance $\boldsymbol{Z}_1$ is connected by two brushes A and B, which may be stationary or rotating, to another impedance $\boldsymbol{Z}_2$ (instead of a simple wire and an ammeter in the classical arrangement of the homo-polar generator) in series with a switch SW, which is controlled remotely from the stationary (laboratory) coordinate system S. The switch SW is assumed to be an idealized one, with the impedance between its two terminals equal to infinity when the switch is open and equal to zero when the switch is closed. The practical switches approximate those conditions only within certain frequency bands.

Let the impedance Z_2, which includes all connecting-wires, and the series switch SW rotate around the z axis with the constant angular velocity $\boldsymbol{\omega}_2 = \omega_2 \mathbf{a}_z$. Since by definition impedances $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ must be time-invariant functions of the complex frequency s only, it is important to note that only one of the two impedances must be an axially symmetrical structure in order to provide circular sliding surfaces for the two brushes A and B. Since it has been already assumed that $\boldsymbol{Z}_1$ is an axially symmetrical structure, impedance Z_2 need not be axially symmetrical. The switch SW is necessary in order to define the zero time, and also to disconnect impedances Z_1 and $\boldsymbol{Z}_2$ at any chosen instant of time. The movement of the moving parts of switch SW is assumed to be strictly radial, so that such a movement does not interfere with the induced electric field due to the circular (rotational) movement of branch Z_2 + SW of the circuit of the homopolar generator. This is a closed circuit by definition, even if impedance Z_1 is that of a coaxial capacitor, which represents a short before it is charged during the transient period.

The above described configuration of the three rotating structures, each rotating with its own constant angular velocity, represents a generalized homopolar generator, In order to avoid the thermal and the contact potential effects, it is assumed that all metallic parts of the circuit of the generalized homopolar generator are made of the same metal, and are at the same temperature. The magnet can be connected to a single reference point of the circuit of the homopolar generator, the ground point, without disturbing the operation of the generalized homopolar generator.

In the forthcoming analysis of the generalized homo-polar generator, the thermal effects, the electromagnetic radiation effects, the thermal noise, the contact potential effects, the inertial effects, and the elastic or plastic deformation effects are assumed to be negligibly small and are neglected.

ANALYSIS OF GENERALIZED HOMOPOLAR GENERATOR

Let us assume that the three angular velocities of the described generalized homopolar generator have reached the steady states. An instrument, a storage, i.e., memory oscilloscope for example, assumed to be insensitive to acceleration and with a very high input impedance is attached to impedance Z_2 in order to observe and record the flow of the electric current through the circuit of the homopolar generator. The instrument is rotating together with the impedance Z_2, It is also possible to form special brushes B′ and B″ instead of a single one, so that the stationary observer can connect an instrument (ammeter) in series with the circuit of the homopolar generator without disturbing the operation of the circuit in any significant way. All three angular velocities, the switch SW, and the instrument are monitored and controlled from the laboratory (stationary) coordinate system S.

Let us close the switch SW at one instant of time, thus defining the zero time for the experiment. The electric current $i = i(t)$ is presumably observed to flow through the circuit of the homopolar generator, due to the induced EMF $u = u(t)$ within that circuit.

Let $I=I(s) = L\{i\}$ denote the Laplace transform of the current flowing through that circuit. Let $U= U(s) = L\{u\}$ be the Laplace transform of the induced EMF. The notations are conventional for the theory of the Laplace transforms. The zero initial conditions of all elements of the circuit of the homopolar generator are assumed. Thus,

$$Z_1 I + Z_2 I = U \quad . \tag{13}$$

represents the Laplace transform of the conventional Kirchhoff voltage law applied to the closed loop formed by impedances Z_1 and Z_2 of the homopolar generator. Note that $u = L^{-1}\{U\}$ is the total induced EMF within that circuit. In order to calculate that induced EMF, the induced electric fields acting within the metallic and/or dielectric parts of those two impedances (branches) must be obtained.

For the axially symmetrical permanent magnet of the homopolar generator, $\partial \mathbf{B}/\partial t \equiv 0$. No free true (already separated) electric charges are assumed anywhere in the space of the homopolar generator. Thus, the electric field acting upon the charges within impedances Z_1 and Z_2 is the well-known motional term $\mathbf{v}\times\mathbf{B}$, which can be directly derived from the general Faraday law of the electromagnetic induction in the integral form $-\,d\Phi/dt$, which is assumed to be valid without any restrictions whatsoever.

In the expression

$$\mathbf{E} = \mathbf{v}\times\mathbf{B} \, , \tag{14}$$

v is the velocity of the moving charge relative to the magnetic induction density field $\mathbf{B}$, i. e., relative to the coordinate system in which that field $\mathbf{B}$ is defined. But the magnetic induction field $\mathbf{B}$ of the axially symmetrical permanent magnet is a peculiar one. It depends on $R= +\,(x^2 + y^2)^{1/2}$ and z only, and not on ϕ by definition. Thus, the lines of force of that field form a continuum which is not discernible mathematically from one coordinate system to the other, if those systems have the common coordinate origin and the common z axis, which is the axis of symmetry of that magnet by assumption. So the question does arise: Which velocity must be used in calculations if the magnet itself rotates? It is in that respect that the moving and the non-moving force line hypotheses have been proposed.

The axially symmetrical permanent magnet (or steady electromagnet) creates a steady magnetic induction density field $\mathbf{B}$ which can be described and/or approximated by using either the scalar potential or the vector potential. Both methods are fully equivalent as long as no true electrical current is encircled in the calculation of the EMF, which is true for the case analyzed here in this paper. The scalar potential method is chosen here since it is somewhat simpler for the mathematical manipulations. The geometrical center of the magnet is assumed to coincide with the origin of the cylindrical coordinate system of reference S. The cylindrical coordinate system is the most appropriate one in this case, in view of the axial symmetry of the problem.

Let the magnetic induction density field $\mathbf{B}$ be represented by

$$\mathbf{B} = -\nabla\Psi = -\frac{\partial \Psi}{\partial R}\mathbf{a}_R - \frac{\partial \Psi}{\partial z}\mathbf{a}_z, \tag{15}$$

where Ψ is the magnetic scalar potential, which is by assumption a well-behaved function of R and z only, i. e., the ϕ dependence is absent by assumption. The notation is conventional.

Since no magnetic monopoles are to be assumed or utilized in this paper, $\nabla \cdot \mathbf{B} \equiv 0$ by definition. This limits the applicability of expression (15) for $\mathbf{B}$ to the region outside the permanent magnet, but includes all points on the surface of the permanent magnet which suffices for the analysis here in this paper. On the other hand, expression (15) can be used in the case of the solenoid-al electromagnet inside as well as outside the solenoid as long as during the calculation of the EMF the closed path of integration never encircles the true electric current. All the above conditions are met in the calculations of the induced EMF in the case of the generalized homopolar generator analyzed in this paper.

In view of $\nabla \cdot \mathbf{B} \equiv 0$, the scalar magnetic potential $\Psi = \Psi(R, z)$ is a harmonic function which satisfies the Laplace equation

$$\nabla^2\Psi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Psi}{\partial R}\right) + \frac{\partial^2 \Psi}{\partial z^2} = 0. \tag{16}$$

Let us assume that the velocity v of the electrical charge rotating within its branch (impedance) is

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} = \omega\mathbf{a}_z \times (R\mathbf{a}_R + z\mathbf{a}_z), \tag{17}$$

where ω is the algebraic value of the constant angular velocity of the rotating charge, to be determined in terms of ω_M, ω_1 and ω_2 depending on the hypothesis assumed, and for which branch of the circuit the velocity Eq. (17) is to be applied.

Introducing Eqs. (15) and (17) into expression (14), $\mathbf{E}$, after some simple vector algebra, becomes

$$\mathbf{E} = -\omega R\frac{\partial \Psi}{\partial z}\mathbf{a}_R + \omega R\frac{\partial \Psi}{\partial R}\mathbf{a}_z. \tag{18}$$

This electric field can also be expressed in the form

$$\mathbf{E} = -\omega\nabla W = -\omega\frac{\partial W}{\partial R}\mathbf{a}_R - \omega\frac{\partial W}{\partial z}a_z, \tag{19}$$

where W= $W(R,z)$ is an appropriate scalar function. Equating the corresponding components from Eqs. (18) and (19), the following equations are obtained:

$$\omega\frac{\partial W}{\partial R} = \omega R\frac{\partial \Psi}{\partial z}, \tag{20}$$

and

Cancelling ω in the above equations and taking the partial derivative of Eq. (20) with respect to z and the partial derivative of Eq. (21) with respect to R, the following expression is obtained

$$R\,\frac{\partial^2\Psi}{\partial z^2}=\frac{\partial^2 W}{\partial z\,\partial R}\equiv\frac{\partial^2 W}{\partial R\,\partial z}=-\frac{\partial}{\partial R}\left(R\,\frac{\partial\Psi}{\partial R}\right). \tag{22}$$

This expression is the identity in view of the Laplace equation [Eq. (16)] for Ψ . Of course the interchange of the order of the partial differentiation of $\Psi(R, z)$ with respect to R and z is assumed to be permissible, which is true under some very broad conditions invariably met in all physical problems.

Thus, it is proven that **E** given by Eq. (18), i. e., Eq. (19), is indeed a conservative electric field under the conditions specified. However, this electric field is not equivalent to a true electrostatic field since this electric field is motional and magnetic according to its origin. It cannot be screened by enveloping closed non-ferromagnetic conductors since it is not a Coulomb-type electrostatic field, which is screenable by nonferro-magnetic conductors according to the experimental evidence. The actual evaluation of $W(R, z)$ requires the detailed specification of the geometrical configuration and either the magnetization density vector or the electric current density vector.

CALCULATION OF EMF

The calculation of the induced EMF within the circuit of the generalized homopolar generator will be done separately for both of the logically possible hypotheses (or theories): The **M** (moving force line) hypothesis (Case 1) and the **N** (nonmoving force line) hypothesis (Case 2).

Case 1

The **M** hypothesis is assumed to be valid in this case. Thus, the magnetic induction density field **B** of the axially symmetrical steady magnet of the generalized homopolar generator rotates together with the magnet with the constant angular velocity $\boldsymbol{\omega}_M$, Consequently, branch Z_1 of the homopolar generator rotates relative to the magnetic induction density field **B** with the constant angular velocity $\boldsymbol{\omega}_1-\boldsymbol{\omega}_M=(\omega_1-\omega_M)\,\mathbf{a}_z$, which is obvious in view of the description and the definition of the generalized homopolar generator. Thus, the induced electric field acting upon the electrical charges within branch Z_1, due to their rotation is obtained by substituting w in the expression (19) with $\omega_1-\omega_M$, which yields

$$\mathbf{E}_{1m}=-\left(\omega_1-\omega_M\right)\nabla W\,, \tag{23}$$

where the index 1 in $\mathbf{E}_{1m}$ denotes that the induced electric field is applicable for branch Z_1; while the second subscript (index) m denotes that the induced electric-field is applicable with the M hypothesis assumed to be valid.

In view of the **M** hypothesis, branch Z_2 rotates with respect to the rotating magnetic induction density field B with the constant angular velocity $\boldsymbol{\omega}_2-\boldsymbol{\omega}_M=(\omega_2-\omega_M)\,\mathbf{a}_z$. Thus, using the expression (19), the induced electric field acting upon the rotating electrical charges within branch Z_2 of the circuit of the generalized homopolar generator is given by

$$\mathbf{E}_{2m} = -(\omega_2 - \omega_M)\,\nabla W\,, \tag{24}$$

where the subscript m has the same meaning as in Eq. (23).

Note that expressions (23) and (24) are valid quite generally for all algebraic values of $\boldsymbol{\omega}_M$, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$, although in the derivations of the relative angular velocities it is most natural to imagine that $\boldsymbol{\omega}_M$ is smaller than both $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$, while all three values are positive.

The induced EMF within the circuit is by definition the strictly closed-loop integral of the electric field around the closed loop. If $\mathbf{E}_m$ denotes formally that electric field, which within the appropriate branches of the circuit is defined by Eqs. (23) or (24), then $u_m = u_m(t)$, *i.e.*, the induced EMF in this Case 1 is given by

$$\begin{aligned} u_m &= \oint \mathbf{E}_m \circ d\mathbf{l} = \int_A^B \mathbf{E}_{2m} \cdot d\mathbf{l} + \int_B^A \mathbf{E}_{1m} \circ d\mathbf{l} \\ &\equiv -(\omega_2 - \omega_M)(W_B - W_A) - (\omega_1 - \omega_M)(W_A - W_B) \\ &\equiv (\omega_1 - \omega_2)(W_B - W_A) \end{aligned} \tag{25}$$

where W_A and W_B denote the values of the potential function $W(R, z)$ at the brush points A and B, respectively. Note that the circulation in the above integration around the closed loop has been performed strictly in the positive mathematical direction.

It is clear that the induced EMF u_m as given by Eq. (25) is independent of the angular velocity $\boldsymbol{\omega}_M$ of the rotating magnet.

Case 2

The N hypothesis is assumed to be valid in this case. In view of this hypothesis, the magnetic induction density field **B** of the axially symmetrical steady magnet of the generalized homopolar generator does not rotate with the rotating magnet by assumption.

Consequently, branch Z_1 of the generalized homo-polar generator rotates relative to the magnetic induction density field **B** with the constant angular velocity $\boldsymbol{\omega}_1 = \omega_1 \mathbf{a}_z$ by definition. Thus, the induced electric field acting upon the electrical charges rotating with branch Z_1 is given by

$$\mathbf{E}_{1n} = -\omega_1 \nabla W\,, \tag{26}$$

where the subscript 1 in $\mathbf{E}_{1n}$ denotes that the induced electric field is applicable for branch Z_1 while the second subscript n denotes that that induced electric field is applicable with the N hypothesis assumed to be valid.

For branch Z_2, which rotates with respect to the magnetic induction density field **B** with the constant angular velocity $\boldsymbol{\omega}_2 = \omega_2 \mathbf{a}_z$ by assumption, the induced electric field is

$$\mathbf{E}_{2n} = -\omega_2 \nabla W\,, \tag{27}$$

where the subscript n has the same meaning as in Eq. (26).

The induced EMF $u_n = u_n(t)$ is given by

$$u_n = \oint \mathbf{E}_n \circ d\mathbf{l} = \int_A^B \mathbf{E}_{2n} \cdot d\mathbf{l} + \int_B^A \mathbf{E}_{1n} \cdot d\mathbf{l}$$
$$\equiv - \omega_2(W_B - W_A) - \omega_1(W_A - W_B)$$
$$\equiv (\omega_1 - \omega_2)(W_B - W_A). \tag{28}$$

The circulation in the above integration around the closed loop has been performed strictly in the positive mathematical direction, i.e., exactly as in Eq. (25).

The comparison of Eqs, (25) and (28) yields

$$u_m \equiv u_n = u \ . \tag{29}$$

Since the above analysis is quite general and applicable to the transient as well as to the steady-state measurements of the current which flows through the circuit of the generalized homopolar generator, which must be considered as closed by definition during the transient periods, the conclusion is reached from the above result *that no experiment with the generalized homopolar generator or its classical form can resolve the puzzle, which one of the two logically possible hypotheses is correct, the moving force line hypothesis or the nonmoving force line hypothesis.*

The above conclusion has been obtained using the integral approach of the circuit theory, which is a branch of the classical electromagnetic field theory. Consequently, the differential approach must yield the same result. However, it is obvious that the differential approach to the solution of this problem is plagued with many difficulties, such as the boundary conditions, the geometrical configurations of branches Z_1 and Z_2, the switch SW, the brushes, etc. The above integral solution has resolved *only* the problem of the flow of the electrical current through the circuit of the generalized homopolar generator, and, consequently, it is possible to calculate the final values of the voltages across impedances Z_1 and Z_2, provided those impedances are known over the entire frequency spectrum with a reasonable accuracy. However, this integral solution cannot and does not give the field distribution within the generalized homopolar generator after the transient period is over.

The preceding analysis clearly shows that the relative rotation of the branches of the circuit of the generalized homopolar generator is essential for the flow of the electrical current through the circuit of that generator. Using this mathematical model of the generalized homopolar generator, which is fully consistent with all experimental results, except for a single result by Pegram,[5] it will be shown that the claim by Kennard, [3,4] that the M hypothesis was disproved, appears to be definitely erroneous. Moreover, if the M hypothesis is to be assumed to have been disproved, then that may be due only to Pegram,[5] who never claimed that, and whose result appears to violate the Faraday law of induction.

BRIEF HISTORICAL REVIEW

A brief historical review is included here in order to appreciate the anguish of many physicists who have attempted to find the solution to the probelm of whether the M or the N hypothesis is correct, contributing in the process to the voluminous and contradictory literature on the subject of the homopolar generator (see Refs. 1-5).

The question about the seat of the EMF of the homo-polar generator in its classical form, which problem is directly connected with the M and N hypothesis, was raised originally by the discoverer of the homopolar generator, i. e., Faraday himself. Faraday expressed vague belief that the seat of the EMF was the rotating magnet, which is an oversimplification of the problem. Thus, Faraday favored the N hypothesis, which belief probably influenced the debate concerning the issue. Faraday experimented only with the classical form of the homopolar generator.

In order to experimentally resolve the puzzle of the M or the N hypothesis, it occurred to Barnett,[1] p. 324, in 1902 (cf. also Fig. 2) "that the problem could be solved by the following method. A cylindrical condenser (capacitor) is placed in an approximately uniform magnetic field parallel to its axis, and the magnets producing the field are rotated while the condenser is short circuited by a wire at rest like itself. While the magnets are still rotating, the connection between the armatures of the condenser is broken; and the condenser is tested for charge after the field is annulled, or the rotation stopped, or the condenser removed. It was argued that if the lines of induction moved with the magnets the condenser would receive charges which could be computed, and that it would remain uncharged in case the lines remained fixed and the magnets moved through them. It was found later that a somewhat similar idea, discussed below, had previously occurred to Preston,[9] and in 1908 Waring[10] proposed an experiment essentially the same in principle as that which occurred to me. As shown below, however, our reasoning was erroneous. Since what precedes was written there has appeared an experimental paper on the subject by Kennard[3] which also is based on incorrect theory."

Barnett argued in his paper[1] that the basic premise of his experiment as well as that of Kennard was wrong. Barnett used rather weak arguments and even some erroneous arguments. Barnett concluded[1] that the experiment could not prove or disprove the M hypothesis. Barnett's experimental results were unmistakable and showed no charge in the condenser (capacitor).

In his second paper[2] and elsewhere, Barnett continued the apparently futile controversy with Kennard, but was obviously unsuccessful. Namely, Barnett was upstaged by Kennard,[3] who only a few months before the publication of the thorough paper[1] by Barnett proclaimed on the basis of a questionable experiment that "thus the moving force line theory is disproved," Kennard's conclusion was apparently wrong, since his, Kennard's as well as Barnett's results can be easily explained equally well on both the M and N hypotheses, as the mathematical model derived in this paper clearly shows. However, Kennard's conclusion was apparently accepted by most physicists as correct.

From the point of view of the mathematical model of the generalized homopolar generator presented in this paper, it is clear that no charge could be found in Barnett's experiment.

Namely, using the notation of this paper, Barnett's experiment involved $Z_1=1/sC_1$, $\omega_1=0$, $Z_2 = r_2$ (very small), and $\omega_2 = 0$. Of course, the impedances can be approximated even better by including the inductive components, but that is absolutely unnecessary. With those values, Eqs. (25) and (28) yield zero for the EMF, regardless of whether the magnet rotates or not. The capacitor cannot be charged, which Barnett found to be true.[1]

Kennard,[3] rotating only a coaxial iron bar inside an electromagnet at rest with the stationary electrometer directly connected to a stationary coaxial capacitor, found no charge and concluded, obviously rather hastily, that "thus the moving force line theory is disproved." Kennard's measuring procedure is obviously identical to Barnett's procedure, but the elimination of the short circuiting simplifies the measurements and provides the direct results.

Without going into the rather heated debate between Barnett and Kennard of how many lines the rotating iron bar carried with the rotation, Kennard's results showed no charge. With the notation used in this paper, Kennard's experiment described in Ref. 3 involved $Z_1 = 1/sC_1$, $\omega_1 = 0$, and $Z_2 = 1/sC_2$, where C_2 is the capacitance of the electrometer and $\omega_2 = 0$, neglecting leakage resistances of the coaxial capacitor C_1 and the electrometer, i. e., the capacitor C_2, as well as the small series resistances and the small self-inductances. The induced EMF is zero according to Eqs. (25) and (28), regardless of whether the magnet or the iron bar rotated or not, carrying or not carrying the magnetic field with the rotation. This confirms Kennard's results reported in Ref. 3.

In order to answer the criticism to his somewhat questionable experiment, Kennard[4] performed a more sophisticated experiment with the possibilities of rotating the electromagnet and the cylindrical coaxial capacitor either together or separately, while the electrometer was always stationary and directly connected to that coaxial capacitor by two brushes (cf. Fig. 2). Kennard[4] found no charge (measured zero voltage by the electrometer) when the electromagnet was rotated, while the cylindrical coaxial capacitor was at rest together with the electrometer, which completely confirmed Barnett's results.[1] Kennard[4] found charge (measured voltage by the electrometer) when the electromagnet and the coaxial capacitor rotated together, while the stationary electrometer was connected to the rotating coaxial capacitor by two appropriate brushes, of which Kennard complained as causing troubles. This, Kennard's result, shows that a coaxial capacitor rotating in the magnetic field is a voltage generator as far as the stationary observer is concerned. Kennard,[4] on the basis of all those unquestionable experimental results, again concluded, but apparently erroneously, that "thus the moving force line theory is disproved. " Kennard in Ref.... 4 and elsewhere continued the rather heated controversy with Barnett.

Kennard's experiment described in Ref. 4 involved, from the point of view of the generalized homopolar generator model $Z_1 = 1/sC_1$, $\omega_1 = 0$ in the first phase of the experiment and ω_1 equal to a finite value during the second phase of the experiment, $Z_2 = 1/sC_2$, where C_2 is the capacitance of the stationary electrometer and $\omega_2 = 0$, neglecting again the large leakage resistances (assuming that they are infinitely large), the small series resistances, and the small self-inductances.

With $\omega_1 \neq 0$ and the switch closed at $t = 0$, Eq. (25) or (28) yields for the EMF $u = \omega_1(W_B - W_A)\, h(t)$, whose Laplace transform is $U = (1/s) \cdot \omega_1(W_B - W_A)$. Note that by definition $1/s = L\{h(t)\}$, where $h(t)$ is the unit step function. Equation (13) yields for the Laplace transform of the current through that circuit

$$I = \frac{C_1 C_2}{C_1 + C_2}\, \omega_1 (W_\mathrm{B} - W_\mathrm{A}) \,. \tag{30}$$

By definition, the Laplace transform $V_2 = V_2(s)$ of the voltage across impedance Z_2 is $V_2 = Z_2 I = I/sC_2$, whose inverse Laplace transform, utilizing Eq. (30), is

$$v_2 = L^{-1}\{V_2\} = \frac{C_1}{C_1 + C_2}\, \omega_1 (W_B - W_A)\, h(t) \,. \tag{31}$$

This is precisely the voltage measured by Kennard using the stationary electrometer during the second phase of the experiment. The rise time is idealized, since the resistances have been neglected.

Actually, assuming that R_A and R_B are the radii of the armatures of the coaxial capacitor with $R_\mathrm{B} > R_\mathrm{A}$, and that the magnetic induction density field **B** within the solenoid used by Kennard,[3,4] Barnett,[1] and also by Pegram[5] is approximately uniform, then the expression (31) can be written approximately

$$v_2 = \frac{C_1}{C_1 + C_2}\, \frac{\omega_1 (R_\mathrm{B}^2 - R_\mathrm{A}^2)\, B}{2}\, h(t) \,. \tag{32}$$

This is the result quoted by Kennard[4] which is fully in accordance with the mathematical model of the generalized homopolar generator, which is presented in this paper.

During the first phase of the experiment, $\omega_1 = 0$, the EMF is zero, and the capacitor cannot be charged.

All those results have been found by Kennard and are reported in Ref. 4. But those results do not disprove the M hypothesis as Kennard has claimed, since they are fully in agreement with the mathematical model of the homopolar generator presented in this paper. Barnett[1] was right that the Barnett-Kennard experiment could not and did not prove or disprove either the M or the N hypothesis. However, Barnett was unable to convince his fellow physicists that he was correct, since Barnett himself believed for a long time that the basic premise behind that Barnett-Kennard experiment was correct. Barnett sensed the error at the last moment through his imagination, which is more important than knowledge, to quote Einstein. Thus, the Barnett-Kennard experiment apparently proved nothing at all, although most physicists, tired of that old and stubborn puzzle, and also excited at that time by the fresh and revolutionary ideas of Bohr, accepted Kennard's conclusion as correct, a view which is held even nowadays by most physicists, notwithstanding some obvious difficulties and inconsistencies (see Ref. 8, p. 363).

It is strange that Kennard did not realize that in his experimental approach the mainly resistive branches of the classical homopolar generator were actually replaced by chiefly capacitive branches, and that the magnetic field of the solenoid of the electromagnet was not confined to the interior space of the solenoid but was also present in the outside space. And the fundamental error appears to be the misconception that an uncharged capacitor during the transient period of the experiment is an open circuit, which led to the apparently erroneous application of Faraday's law of electromagnetic induction without closing the loop of integration during the calculation of the EMF, and it takes the EMF to charge a capacitor.

It must be mentioned here that apparently a number of other renowned scientists, beside Barnett himself, such as Poincare, Abraham, Hertz, etc., also held the view that the measurements with the open circuit of the homopolar generator in the Barnett-Kennard experiment could not resolve the puzzle whether the M or the N hypothesis is correct. However, unfortunately, neither Barnett nor anybody else so far has put forward strong and correct arguments against Kennard's erroneous conclusion that "thus the moving force line theory is disproved." Barnett came most closely to the truth, although he actually sensed the error at the last moment.

As a matter of fact, the confusion about this problem in the published literature is so great that even some experimental results are claimed to have been measured, although those experimental results obviously violate the Faraday induction law, even for the conventionally closed circuit (cf. Ref. 4, p. 180). In that respect it must be mentioned that Pegram[5] reported experimental results which violate Faraday's law. Namely, Pegram[5] rotated the coaxial capacitor inside an electromagnet together with the rod, which short circuited the capacitor for a moment during the rotation, in order to charge that capacitor. Clearly, since $\omega_1 = \omega_2$ for such a case, the EMF is identically equal to zero according to Eq. (25) or (28), regardless of whether the magnet rotates or not, and the capacitor cannot be charged. Pegram, however, reported to have measured the charge within that capacitor, which appears to be the violation of the Faraday induction law and, as a consequence, the violation of the principle of the conservation of energy. Pegram's result is definitely at variance with the mathematical model of the generalized homopolar generator presented in this paper.

However, at the time of his experiment, Pegram already believed that the M hypothesis was disproved by Kennard, and so Pegram expected that result, which he claimed to have measured. Pegram's description in Ref. 5 casts doubts over his results. One possible explanation of Pegram's result to have found charge in that case is that Pegram has actually stopped the short-circuiting rod for a very short period of time during the opening process, Pegram used some stationary rod to push open the short-circuiting rotating rod instead of securing a strict radial opening of that short-circuiting rod without any circular component of velocity added or substracted from the actual rotating velocity of that rod. Indeed, such a short stopping of the short-circuiting rotating rod is almost inevitable unless very special precautions are made. Even if the time interval of stopping the short-circuiting rod was of the order of 1 nsec. of which possibility Pegram was apparently unaware and the measurements of which were impossible during Pegram's lifetime, but such a short stopping must be considered as inevitable due to the inertia, then in view of the estimated capacitance of the order of 100 pF and the estimated resistance of the short-circuiting rod of the order of 10 mΩ thus yielding the time constant of the circuit of the order of 1 psec, the capacitor must have been charged almost inexorably. Thus, Pegram's result violates the Faraday induction law, and it appears to favor the N hypothesis, which Pegram believed to have been proved by Kennard. Thus, it is only Pegram who could have claimed to have disproved the M hypothesis, which he never did. Pegram also reported to have performed the experiment with the stationary capacitor and the stationary short-circuiting rod while the magnet rotated, confirming Barnett's result, Pegram's report about that part of the experiment is contained in a single sentence, without any numerical values at all, which is strange and suggests that Pegram was not very careful. As a matter of fact, Pegram's paper appeared when the physicists of that bygone era no longer cared about that problem, since the problem has been already considered as "solved" and "closed." It is very probable that nobody read Pegram's paper[5] very carefully at that time.

Actually, Pegram's result not only contradicts the mathematical model presented in this paper, but it also contradicts Kennard's unquestionable result,[4] as well as that of many other physicists (cf. Ref. 2, p. 325), i.e., that a rotating uncharged coaxial capacitor in the magnetic field is observed by the stationary observer as a source of the voltage which, using the same notations and approximations as for the expression (32), can be written in the form

$$u_1 = \tfrac{1}{2}\omega_1 B(R_B^2 - R_A^2)\, h(t)\,. \tag{33}$$

Thus, in view of the generalized Thevenin theorem, the rotating coaxial uncharged capacitor in the magnetic field is equivalent to the voltage generator, whose voltage u_1 is given by Eq. (33) and is measured across the stationary brushes **A** and **B**, i.e., $L\{u_1\}$, in series with the capacitive internal impedance $1/sC_1$. The Laplace transform notation is used.

On the other hand, the rotating metallic rod is fully equivalent to and can be replaced by a rotating disk, provided the attachments, i.e., brush points, remain the same. Assuming R_A and R_B as the radii of the brushes **A** and **B**, the voltage of the disk (Faraday disk) rotating with the constant angular velocity $\boldsymbol{\omega}_2$ in the approximately uniform magnetic field $\mathbf{B}$, as used for Eqs. (32) and (33), is given by

$$u_2 = \tfrac{1}{2}\omega_2 B(R_B^2 - R_A^2)\, h(t)\,. \tag{34}$$

Thus, in view of the generalized Thevenin theorem, the rotating disk, or rod to that matter, is equivalent to the voltage generator, whose voltage u_2 is given by Eq. (34), which is measured across the stationary brushes A and B, in series with the small internal resistance r_2.

Note that the polarities of u_1 and u_2 are such that they are in opposition. When $\boldsymbol{\omega}_1 \equiv \boldsymbol{\omega}_2$, i.e., when the capacitor and the disk or rod are rotated together, while the brushes are stationary, or even removed as unnecessary, as Pegram did in his experiment, the voltages u_1 and u_2 cancel each other, and no electrical current observed by the stationary observer can flow, which may result in the separation of the mobile charges. Consequently, the coaxial capacitor could not have been charged, as reported by Pegram, i.e., Pegram's result appears to have been an error.

Actually, Pegram's result implies that either the coaxial rotating capacitor or the rotating rod, i. e., disk, is not the source of the potential difference, i. e., voltage. But that contradicts experimental results by many physicists, including Kennard.[4] A rotating coaxial capacitor in the axial magnetic field is definitely a voltage generator with the finite capacity for the delivery of the electrical charge, due to its capacitive internal impedance. The Faraday disk or the rotating rod in the axial magnetic field is also a voltage generator. Those two voltages in Pegram's experiment cancel each other according to the mathematical model presented in this paper. Pegram's claimed result implies that it is irrelevant whether one of those two structures is rotating or not. Carried even further, that would imply that one structure could rotate in the opposite direction or with any arbitrary angular velocity without changing the result, i. e., the result remains the same under arbitrarily large numbers of distinctly different physical conditions. That appears to be absurd, although it is logically possible but very improbable. Thus, Pegram's result appears clearly to be an error, which violates the Faraday law of electromagnetic induction.

CONCLUSION

It has been shown that the admissible circular trajectory of a test point electrical charge in the equatorial plane of the fixed point magnetic dipole can be interpreted by the stationary observer as the revolution of the test point charge around another electrical point charge. On the other hand, for the observer attached to the test point charge, it appears that the point magnetic dipole is revolving around that observer, and at the same time the magnetic dipole is spinning with the synchronous angular velocity in the negative direction. This dynamic problem suggests that a spinning time-invariant point magnetic dipole may be considered as a possible model of the electrical charge within the framework of classical physics, provided the M hypothesis was not disproved, as claimed by Kennard.[3,4] but contravened unsuccessfully by Barnett.[1,2]

In view of the potentially high significance of the quantum-mechanical electrical charge model proposed recently and extensively analyzed by Jehle, [6,7] and in view of the intriguing possibility of the electrical charge model within the framework of classical physics as exposed above, which is basically similar to Jehle's model, the controversy at the turn of this century concerning the moving or the nonmoving force line theory was reanalyzed.

That analysis reveals the fact that if the mathematical model of the generalized homopolar generator, derived in this paper, is used, then the claim by Kennard that "thus the moving force line theory is disproved" must be considered as incorrect, since all results by Barnett, Kennard, and others, except for a single result by Pegram,[5] can be explained on either the M or the N hypothesis. Pegram's result might be argued as disproving the M hypothesis, but that result also violates the Faraday law of induction and it appears to be an error. Pegram believed when he made that experiment that the M hypothesis was already disproved. *But the M hypothesis was not disproved as shown, and it appears definitely to be more logical and preferable, unless the concept of the field is a pure abstraction, devoid of any physical meaning.*

Further experimental research with all kinds of spinning magnets, including the repetition of Pegram's experiment, appears to be very interesting from the point of view of search for truth, with a view towards an intriguing model of the electrical charge. The controversy at the turn of this century has apparently not resolved the old but stubborn puzzle of the moving or the non-moving force line theory which, to the best of imagination and knowledge of this author, may prove, after all, to be an antinomy or the breakdown of the classical electromagnetic field theory.

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