GEOPHYSICAL COORDINATE SYSTEM

by
Jovan Djuric, retired UNM professor
Balkanska 28, 11000 Belgrade, Serbia
E-mail: oliverdj@eunet.rs

PACS: 01.30.M or 01.30.Os

ABSTRACT

This short paper deals with the peculiar approximation problem in geophysics, that in the presently exclusively used geophysical inadequate coordinate system, whose coordinate origin is defined to be at the center of mass of the Earth, the International Gravity Formula must be used with one flattening for the Northern hemisphere and the different flattening for the Southern hemisphere. It is suggested in this paper, that a new geophysical coordinate system should be defined, so that a new gravity formula in reference to that new geophysical coordinate system may contain in itself the obvious additional dependence of the Earth's gravity on the geographic latitude, and thus be applicable for the entire Earth without any reference to the different flattenings. The new geophysical coordinate system is proposed in this paper. The similar coordinate systems must be used in all planetary, solar and stellar problems.

INTRODUCTION

The International Gravity Formula as defined in the presently exclusively used geophysical coordinate system is given by the expression (see [1], p. 79)

$$g = 9.780490(1 + 0.0052884\sin^2\lambda - 0.0000059\sin^2 2\lambda) \quad m/s^2 \quad , \qquad (1)$$

where λ is the geographic latitude. It is obvious that this Formula contains only the monopolar term of the Earth's gravity. The second term in the parenthesis is due to the rotation of the Earth, while the third term is assumed for fitting. This Formula (1) does not show explicitly the dependence on r and φ, but the flattening $f^{-1} = 298.5$ must be used for the Northern hemisphere, and the flattening $f^{-1} = 297.3$ for the Southern hemisphere (v. [1], p. 79). This fact suggests that some form of the dipolar term of the Earth's gravity should be present in the generally applicable Earth's gravity formula.

GENERAL EXPRESSION OF THE EARTH'S GRAVITY

The Earth's gravitational potential is given in the general coordinate system is given by the expression (see [2], or any comprehensive mechanics textbook)

$$U_E = G\iiint \frac{\rho_{VE}\, dV'}{\left|\vec{r} - \vec{r}\,'\right|} \quad , \qquad (2)$$

where $G = 6.67\times 10^{-11} kg^{-1} m^3 s^{-2}$ is the gravitational constant , ρ_{VE} is the volume mass distribution of the Earth and the vector $\vec{r}$ defines the point of observation. Retaining only two terms in the Taylor development, it becomes

$$U_E = \frac{GM_E}{|\vec{r}|} + \frac{G\vec{M}_{1E} \cdot \vec{r}}{|\vec{r}|^3} \quad . \qquad (3)$$

M_E is the mass of the Earth, while $\vec{M}_{1E}$ is the first mass moment of the Earth, also called the dipolar moment of the Earth.. They are given by the expressions

$$M_E = \iiint \rho_{VE} dV' \quad , \qquad (4)$$

and

$$\vec{M}_{1E} = \iiint \rho_{VE} \vec{r}' dV' \quad . \qquad (5)$$

It should be emphasized at this point that the next term in the Taylor development of the Earth's gravitational potential is divided by the radius r , and in view of the Earth's radius of 6.37×10^6 m, its contribution on the surface of the Earth is quite negligible.

The Earth's gravitational field is by definition

$$\vec{g}_E = -\nabla U_E = \frac{GM_E \vec{r}}{|\vec{r}|^3} - \frac{G\vec{M}_{1E}}{|\vec{r}|^3} + \frac{3G(\vec{M}_{1E} \cdot \vec{r})\vec{r}}{|\vec{r}|^5} \quad . \qquad (6)$$

DISCUSSION

In the presently used geophysical coordinate system the first mass moment of the Earth is by definition zero, and the approximate gravitational potential of the Earth contains only one term, the first monopolar term in the Equation (3), and only the first monopolar term in the Equation (6) fir the Earth's gravitational field. In fact, the International Gravity Formula (1) is obtained by using only the first monopolar term of the Earth's gravitational field in the Equation (6).

As mentioned in connection with the International Gravity Formula, the flattening for the Northern hemisphere is different from the flattening for the Southern hemisphere, which implies that the so-called dipolar term due to the mass moment must be used in order to obtain the correct general gravity formula. The two additional terms in the Equation (6) are due to the obvious Earth's mass moment. In view of the form of the Earth and depending on the Earth's first mass moment, it must contain a horizontal component beside the radial component, which is very interesting. The third term in (6) is only radial, just as the first monopolar term.

The only way to take into account the first Earth's mass moment in the expression for the Earth's gravitational field is to define a new geophysical coordinate system whose coordinate origin is not the center of mass of the Earth. Indeed, the conventional geophysical coordinate system is quite peculiar, since its coordinate origin - the center of mass of the Earth - cannot be determined experimentally even in principle, since its experimental determination necessitates the strictly uniform external gravitational field stretching infinitely, and such a field does not exist in nature. The inaccuracy of the position of the coordinate origin of the conventional geophysical coordinate system of only 10 meters, which is very small compared to the Earth's radius and quite plausible and probable, such inaccuracy changes the 5th decimal figure in the IGF, Equation (1).

It is quite clear that the presently used geophysical coordinate system is very inadequate for the accuracy, and it excludes totally without any justification whatsoever the impact and the effect of the first mass moment in the various physics problems, particularly the gravitational problems, by the arbitrary and quite inappropriate choice of its coordinate origin. A new geophysical coordinate system must be defined.

PROPOSAL

It is obvious that the only point inside the Earth, which has a physical meaning beside the Earth's center of mass, is the point at which the Earth's gravitational field is zero. Such a point certainly exists inside the Earth and can be determined experimentally in principle. That point does not coincide with the Earth's center of mass. Namely, the center of mass of the Earth is defined in the general coordinate system by the expression

$$\vec{r}_{cmE} = \frac{1}{M_E} \iiint \rho_{VE} \vec{r}\,' dV' \quad , \tag{7}$$

while the point at which the Earth's gravitational field is zero, named the center of gravitation of the Earth $\vec{r}_{sgE}$ is the solution of the integral equation

$$\vec{g}_E(\vec{r}_{cgE}) = 0 = G \iiint \frac{\rho_{VE}(\vec{r}_{cgE} - \vec{r}\,')dV'}{\left|\vec{r}_{cgE} - \vec{r}\,'\right|^3} = 0 \quad . \tag{8}$$

It is clear from these two Equations (7) and (8) that the center of mass of the Earth and the so-called center of gravitation of the Earth are two distinctly different points, except in the case of the absolute perfect symmetry of the Earth, which is not the case.

The first mass moment of the Earth with respect to the center of gravitation of the Earth should be called the intrinsic mass moment of the Earth, since that first mass moment of the Earth is absolutely unique and quite significant as the measure of the deviation of the form of the Earth from the perfect sphere.

It must be emphasized at the end of this paragraph that the similar coordinate systems with the coordinate origins at the respective centers of gravitation of the observed mass objects must be used in all planetary, solar and stellar problems,

CONCLUSION

It is obvious that there is a serious problem of approximation in geophysics, since the conventional geophysical coordinate system with its coordinate origin at the Earth's center of mass is definitely inadequate. The above proposed new geophysical coordinate system with its coordinate origin at the center of gravitation of the Earth should resolve that problem. The general gravity formula for the Earth must include beside the Equation (6) the centrifugal acceleration due to the rotation of the Earth. The determination of the new general gravity formula must be the subject of another paper.

REFERENCES

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