CENTER OF MASS AND CENTER OF GRAVITATION

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PACS: 01.50.Zv

ABSTRACT

The center of mass and the center of gravitation of a mass distribution are strictly defined. Particularly, the=center of gravitation is the point at which the self gravitation of a mass distribution is zero. It is shown further that they are the two distinctly different points which coincide if and only if the mass distribution is symmetrical. The center of gravity, which is defined in about half of the available textbooks, but completely and **correctly** eliminated in many, about one half of the available textbooks, is in fact the center of mass, i.e., the center of weight, and it is **not at all** the center of gravitation as defined in this paper, although *gravity* and *gravitation* are considered as the synonyms in all standard English dictionaries.

INTRODUCTION

The center of mass of a mass distribution is invariably defined in all textbooks on mechanics and physics as the point with respect to which the mass moment of that mass distribution is zero. On the other hand, the center of gravitation (or gravity) is and must be logically defined as the point at which the gravitation, or gravity, or the gravitational field of that mass distribution is exactly equal to zero. Note that gravitation and gravity are the two fully interchangeable words with the identical meanings according to any modern edition of the Webster College Dictionary, but *center of gravitation* rather than *center of gravity* is used in this paper for emphasis and to underline the distinction with the word usage so far. But, it is strange to find in some, even many textbooks that those two points, i.e., the center of mass and the center of gravitation (or gravity as used in those textbooks, while gravitation is used in this paper to emphasize the difference as stated above) are considered as the one single point.

PROBLEM

This confusion was obviously caused by some historical linguistic inconsistencies. Namely, the modern Latin was almost exclusively used in science during the 16^{th}, 17^{th} and 18^{th} centuries. The Latin word *gravitas* was translated in English as *gravity*, rather than the simple English word *weight*. Thus, the modern Latin expression *centrum gravitatis* (*gravitatis* is the genitive case of the Latin word *gravitas*) was translated in English as *center of gravity*, which was quite correct at one time, with the simple meaning in straightforward English the center of weight. This point is obviously also the center of mass of that mass distribution in the strictly uniform gravitational field. The Earth's gravitational field is approximately uniform within the confines of a laboratory room. It is also obvious that the expression *center of mass* is the exclusive, intrinsic property of a mass distribution without any reference to any gravitational field, while *center of gravity* in fact *center of weight* implies the presence of an external

gravitational field which must be defined in order to define that expression quite clearly and exactly.

So, there is no doubt whatsoever that at one time in history *center of mass* and *center of gravity* meant the same single point of a mass distribution. The term *specific gravity*, which is used without any problems up to now, is the testimony of that usage. But language is the living tool for communication and it changes in time. During the 17th, 18th and 19th centuries the term *gravity* assumed an entirely different meaning, particularly in the expression *Newton's law of gravity*, in which expression *gravity* does not mean *weight*, but rather *gravitation*, or *gravitational field,* while in the term *specific gravity,* it still means *weight*. Consequently, the expression *center of gravity* no longer means what originally it meant, but rather the center of the gravitation, or the gravitational field, which is zero at that point. Thus, the term *center of gravity, i.e., center of gravitation* (as used in this paper for emphasis) must be defined as the point where the gravitational field of the observed mass distribution is zero, as clearly stated in the first paragraph of this short paper. These two points of a mass distribution *center of mass* and *center of gravitation* are the two quite distinctly different points, and they are obviously invariant if the mass distribution is invariant. These two points coincide if and only if the mass distribution is symmetrical. It must be emphasized that the textbooks, which eliminated the center of gravity, i.e., center of weight in fact, are quite right, if gravity is assumed to mean only weight, which assumption is not quite correct.

As an illustrative example, consider the simplest mass distribution consisting of the two point masses m_1 and m_2 at a distance d from each other. Let d_{cm} designate the distance of the center of mass measured from the mass point m_1 along the line connecting these two point masses, then $m_1 d_{cm} = m_2 (d - d_{cm})$, which yields $d_{cm} = dm_2 / (m_1 + m_2)$. On the other hand, let d_{cg} designate the distance of the center of gravitation measured from the same point mass m_1 along the line connecting these two point masses, then applying the Newton's law of gravity we write $Gm_1 / {d_{cg}}^2 = Gm_2 / (d - d_{cg})^2$, which yields the expression $d_{cg} = d\sqrt{m_1} / (\sqrt{m_1} + \sqrt{m_2})$. G is, of course, the universal gravitational constant. For $m_1 > m_2$, it is easily proved that $d_{cg} > d_{cm}$. It is obvious from the obtained expressions that these two centers coincide, if and only if those two point masses become equal, in which case that simplest mass distribution becomes symmetrical evidently.

CONCLUSION

This author is a retired university professor and has no access to the current editions of the textbooks on mechanics, but he is advised that about one half of the current editions define correctly only the center of mass without even mentioning the center of gravitation (or gravity), while the other half of the current editions still define the center of gravity (the word *gravity* is always used in those textbooks, not the word *gravitation*) as being the same point as the center of mass, which is obviously confusing

and wrong nowadays as stated in this paper, and such center of gravity or weight in fact, is absolutely unnecessary, i.e., superfluous..

It appears that **not a single** of the currently available textbooks on mechanics in English defines the center of gravitation (or gravity, or gravitational field) properly, as defined above in this paper, and as it must be defined logically and correctly as the point of a mass distribution at which the gravitation (or gravity or gravitational field) of that mass distribution is zero. That point, i.e., the center of gravitation must exist as a unique point for any mass distribution, and it is obviously an important point of the observed mass distribution.

This obvious disharmony and the obvious deficiency in the textbooks should be certainly discussed and must be eliminated. It is strange that this obvious problem was allowed to continue in the physics literature for centuries apparently

It must be emphasized that the center of mass as defined everywhere in the physics literature and the center of gravitation as defined in this paper, i.e., as the point at which the gravitation of a mass distribution is zero, are the two distinctly different and very important points of a mass distribution which coincide if and only if that mass distribution is strictly symmetrical which almost never occurs in nature, since the inexorable presence of an external gravitational field makes the space essentially anisotropic, and that makes the exact symmetry of a mass distribution almost impossible, except in some very special situations.